(3) $(\tan^{-1}x)' = \dfrac{1}{1+x^2}$, $(-\infty < x < \infty)$

(4) $(\csc^{-1}x)' = -\dfrac{1}{|x|\sqrt{x^2-1}}$, $(|x| > 1)$

(5) $(\sec^{-1}x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$, $(|x| > 1)$

(6) $(\cot^{-1}x)' = -\dfrac{1}{1+x^2}$, $(-\infty < x < \infty)$

예제 5 함수 $y = \tan^{-1}x$ 의 도함수를 구하여라.

풀이 역삼각함수의 정의에 의해 $x = \tan y$ 이므로 음함수 미분법을 이용하면

$1 = \sec^2 y \cdot \dfrac{dy}{dx}$ 이고 $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$ 이므로

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2} \quad (-\infty < x < \infty)$$

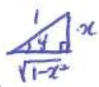
답 $\dfrac{1}{1+x^2}$

문제 5 다음 함수를 미분하여라.

(1) $y = \cos^{-1}(1-2x)$ (2) $y = \cot^{-1}\left(\dfrac{1-x}{1+x}\right)$

[사고력 문제] 1 $\cos(\sin^{-1}x) = \sqrt{1-x^2}$ 임을 보여라.

let $y = \sin^{-1}x$

$\Rightarrow x = \sin y \Rightarrow$ 

$\therefore \cos(\sin^{-1}x) = \cos y = \sqrt{1-x^2}$

[사고력 문제] 2 $\tan^{-1}x + \cot^{-1}x = \dfrac{\pi}{2}$ 임을 보여라.

let $f(x) = \tan^{-1}x + \cot^{-1}x$

$\Rightarrow f'(x) = \dfrac{1}{1+x^2} - \dfrac{1}{1+x^2} = 0$

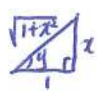
$\therefore f(x) = c$

$f(1) = \tan^{-1}(1) + \cot^{-1}(1) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$\therefore f(x) = \frac{\pi}{2}$

[사고력 문제] 3 $\cos(\tan^{-1}x) = \dfrac{1}{\sqrt{1+x^2}}$ 임을 보여라.

let $y = \tan^{-1}x$

$\Rightarrow x = \tan y$ 

$\therefore \cos(\tan^{-1}x) = \cos y = \dfrac{1}{\sqrt{1+x^2}}$

[문제 해결력 문제] $f(x) = x\sin^{-1}\left(\dfrac{x}{4}\right) + \sqrt{16-x^2}$ 일 때 $f'(2)$를 구하여라.

$f'(x) = \sin^{-1}\frac{x}{4} + x\dfrac{\frac{1}{4}}{\sqrt{1-(\frac{x}{4})^2}} + \dfrac{-2x}{2\sqrt{16-x^2}}$

$\therefore f'(2) = \sin^{-1}\frac{1}{2} + 2\dfrac{\frac{1}{4}}{\sqrt{\frac{3}{4}}} + \dfrac{-2}{2\sqrt{3}}$

$= \frac{\pi}{6} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{\pi}{6}$